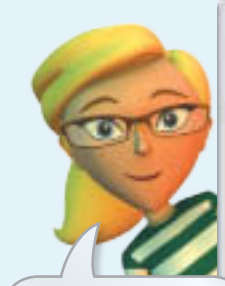
2-3 Biconditionals and Definitions

Common Core State Standards

Prepares for G-CO.C.9 Prove theorems about lines and angles. **Also Prepares for G-CO.C.10, G-CO.C.11**

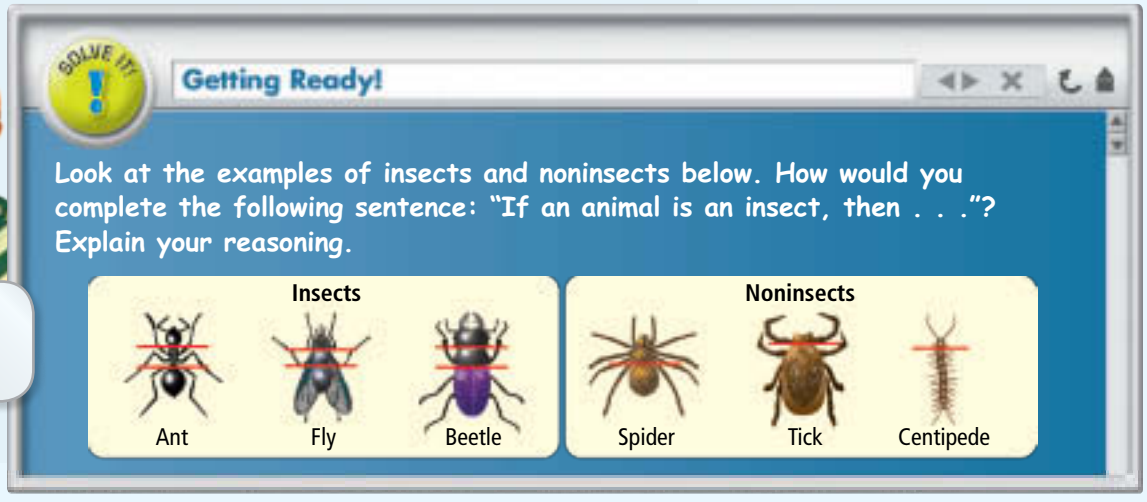
MP 3, MP 6, MP 7

Objective To write biconditionals and recognize good definitions



Make sure you consider all the data.

MATHEMATICAL PRACTICES



Lesson Vocabulary

- biconditional

In the Solve It, you used conditional statements. A **biconditional** is a single true statement that combines a true conditional and its true converse. You can write a biconditional by joining the two parts of each conditional with the phrase *if and only if*.

Essential Understanding A definition is good if it can be written as a biconditional.

Problem 1 Writing a Biconditional

What is the converse of the following true conditional? If the converse is also true, rewrite the statements as a biconditional.

If the sum of the measures of two angles is 180, then the two angles are supplementary.

Converse: If two angles are supplementary, then the sum of the measures of the two angles is 180.

The converse is true. You can form a true biconditional by joining the true conditional and the true converse with the phrase *if and only if*.

Biconditional: Two angles are supplementary if and only if the sum of the measures of the two angles is 180.

Think

How else can you write the biconditional?

You can also write the biconditional as "The sum of the measures of two angles is 180 if and only if the two angles are supplementary."

Got It? **1.** What is the converse of the following true conditional? If the converse is also true, rewrite the statements as a biconditional.

If two angles have equal measure, then the angles are congruent.

take note

Key Concept Biconditional Statements

A biconditional combines $p \rightarrow q$ and $q \rightarrow p$ as $p \leftrightarrow q$.

Example	Symbols	How to Read It
A point is a midpoint if and only if it divides a segment into two congruent segments.	$p \leftrightarrow q$	"p if and only if q"

You can write a biconditional as two conditionals that are converses.

Problem 2 Identifying the Conditionals in a Biconditional

Plan

How can you separate the biconditional into two parts?
Identify the part before and the part after the phrase *if and only if*.

What are the two conditional statements that form this biconditional?

A ray is an angle bisector if and only if it divides an angle into two congruent angles.

Let p and q represent the following:

p: A ray is an angle bisector.

q: A ray divides an angle into two congruent angles.

$p \rightarrow q$: If a ray is an angle bisector, then it divides an angle into two congruent angles.

$q \rightarrow p$: If a ray divides an angle into two congruent angles, then it is an angle bisector.

Got It? **2.** What are the two conditionals that form this biconditional?
Two numbers are reciprocals if and only if their product is 1.

As you learned in Lesson 1-2, undefined terms such as *point, line,* and *plane* are the building blocks of geometry. You understand the meanings of these terms intuitively. Then you use them to define other terms such as *segment*.

A good definition is a statement that can help you identify or classify an object. A good definition has several important components.

- ✔ A good definition uses clearly understood terms. These terms should be commonly understood or already defined.
- ✔ A good definition is precise. Good definitions avoid words such as *large, sort of,* and *almost*.
- ✔ A good definition is reversible. That means you can write a good definition as a true biconditional.

Problem 3 Writing a Definition as a Biconditional

Plan

How do you determine whether a definition is reversible?
Write the definition as a conditional and the converse of the conditional. If both are true, the definition is reversible.

Is this definition of *quadrilateral* reversible? If yes, write it as a true biconditional.
Definition: A quadrilateral is a polygon with four sides.

Think	Write
Write a conditional.	**Conditional:** If a figure is a quadrilateral, then it is a polygon with four sides.
Write the converse.	**Converse:** If a figure is a polygon with four sides, then it is a quadrilateral.
The conditional and its converse are both true. The definition is reversible. Write the conditional and its converse as a true biconditional.	**Biconditional:** A figure is a quadrilateral if and only if it is a polygon with four sides.

Got It? **3.** Is this definition of *straight angle* reversible? If yes, write it as a true biconditional.
A straight angle is an angle that measures 180.

One way to show that a statement is *not* a good definition is to find a counterexample.

Problem 4 Identifying Good Definitions

Plan

How can you eliminate answer choices?
You can eliminate an answer choice if the definition fails to meet any one of the components of a good definition.

Multiple Choice **Which of the following is a good definition?**

(A) A fish is an animal that swims.
(B) Rectangles have four corners.
(C) Giraffes are animals with very long necks.
(D) A penny is a coin worth one cent.

Choice A is not reversible. A whale is a counterexample. A whale is an animal that swims, but it is a mammal, not a fish. In Choice B, *corners* is not clearly defined. All quadrilaterals have four corners. In Choice C, *very long* is not precise. Also, Choice C is not reversible because ostriches also have long necks. Choice D is a good definition. It is reversible, and all of the terms in the definition are clearly defined and precise. The answer is D.

Got It? **4. a.** Is the following statement a good definition? Explain.
A square is a figure with four right angles.
b. Reasoning How can you rewrite the statement "Obtuse angles have greater measures than acute angles" so that it is a good definition?

Lesson Check

Do you know HOW?

1. How can you write the following statement as two true conditionals?

Collinear points are points that lie on the same line.

2. How can you combine the following statements as a biconditional?

If this month is June, then next month is July.
If next month is July, then this month is June.

3. Write the following definition as a biconditional.

Vertical angles are two angles whose sides are opposite rays.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Explain how the term *biconditional* is fitting for a statement composed of *two* conditionals.

5. Error Analysis Why is the following statement a poor definition?

Elephants are gigantic animals.

6. Compare and Contrast Which of the following statements is a better definition of a linear pair? Explain.

A linear pair is a pair of supplementary angles.

A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Each conditional statement below is true. Write its converse. If the converse is also true, combine the statements as a biconditional.

See Problem 1.

7. If two segments have the same length, then they are congruent.

8. Algebra If $x = 12$, then $2x - 5 = 19$.

9. If a number is divisible by 20, then it is even.

10. Algebra If $x = 3$, then $|x| = 3$.

11. In the United States, if it is July 4, then it is Independence Day.

12. If $p \rightarrow q$ is true, then $\sim q \rightarrow \sim p$ is true.

Write the two statements that form each biconditional.

See Problem 2.

13. A line bisects a segment if and only if the line intersects the segment only at its midpoint.

14. An integer is divisible by 100 if and only if its last two digits are zeros.

15. You live in Washington, D.C., if and only if you live in the capital of the United States.

16. A polygon is a triangle if and only if it has exactly three sides.

17. An angle is a right angle if and only if it measures 90.

18. Algebra $x^2 = 144$ if and only if $x = 12$ or $x = -12$.

Test each statement below to see if it is reversible. If so, write it as a true biconditional. If not, write *not reversible.*

See Problem 3.

19. A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to a segment at its midpoint.

20. Complementary angles are two angles with measures that have a sum of 90.

21. A Tarheel is a person who was born in North Carolina.

22. A rectangle is a four-sided figure with at least one right angle.

23. Two angles that form a linear pair are adjacent.

Is each statement below a good definition? If not, explain.

See Problem 4.

24. A cat is an animal with whiskers.

25. The red wolf is an endangered animal.

26. A segment is part of a line.

27. A compass is a geometric tool.

28. Opposite rays are two rays that share the same endpoint.

29. Perpendicular lines are two lines that intersect to form right angles.

B Apply

30. Think About a Plan Is the following a good definition? Explain.

A ligament is a band of tough tissue connecting bones or holding organs in place.

- Can you write the statement as two true conditionals?
- Are the two true conditionals converses of each other?

31. Reasoning Is the following a good definition? Explain.

An obtuse angle is an angle with measure greater than 90.

32. Open-Ended Choose a definition from a dictionary or from a glossary. Explain what makes the statement a good definition.

33. Error Analysis Your friend defines a right angle as an angle that is greater than an acute angle. Use a biconditional to show that this is not a good definition.

34. Which conditional and its converse form a true biconditional?

(A) If $x > 0$, then $|x| > 0$.

(B) If $x = 3$, then $x^2 = 9$.

(C) If $x^3 = 5$, then $x = 125$.

(D) If $x = 19$, then $2x - 3 = 35$.

Write each statement as a biconditional.

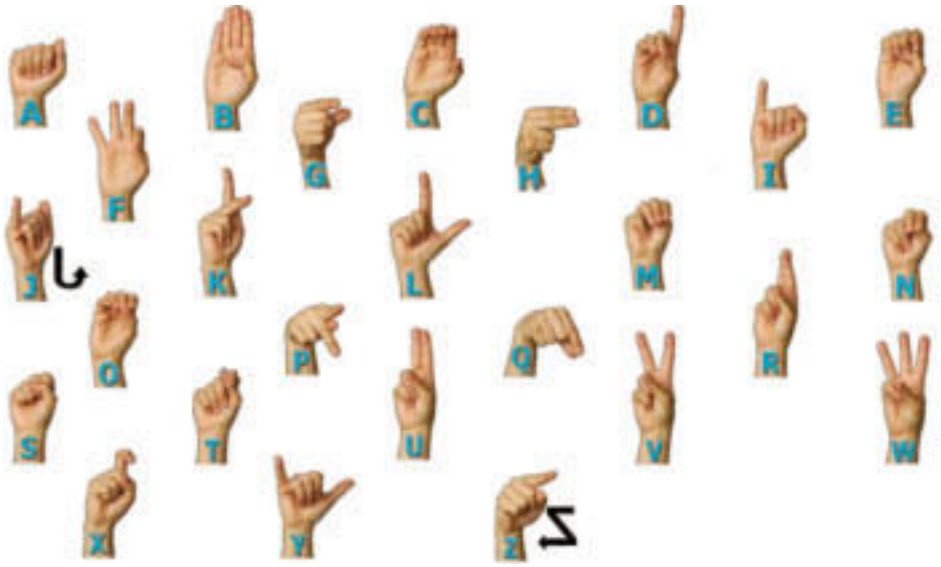
35. Points in Quadrant III have two negative coordinates.

36. When the sum of the digits of an integer is divisible by 9, the integer is divisible by 9 and vice versa.

37. The whole numbers are the nonnegative integers.

38. A hexagon is a six-sided polygon.

Language **For Exercises 39–42, use the chart below. Decide whether the description of each letter is a good definition. If not, provide a counterexample by giving another letter that could fit the definition.**



39. The letter *D* is formed by pointing straight up with the finger beside the thumb and folding the other fingers and the thumb so that they all touch.

40. The letter *K* is formed by making a *V* with the two fingers beside the thumb.

41. You have formed the letter *I* if and only if the smallest finger is sticking up and the other fingers are folded into the palm of your hand with your thumb folded over them and your hand is held still.

42. You form the letter *B* by holding all four fingers tightly together and pointing them straight up while your thumb is folded into the palm of your hand.

Reading Math **Let statements *p*, *q*, *r*, and *s* be as follows:**

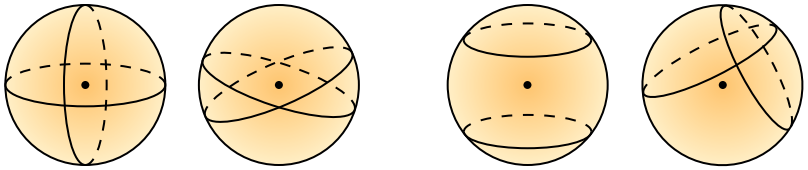
p: $\angle A$ and $\angle B$ are a linear pair.
q: $\angle A$ and $\angle B$ are supplementary angles.
r: $\angle A$ and $\angle B$ are adjacent angles.
s: $\angle A$ and $\angle B$ are adjacent and supplementary angles.

Substitute for *p*, *q*, *r*, and *s*, and write each statement the way you would read it.

43. $p \rightarrow q$ **44.** $p \rightarrow r$ **45.** $p \rightarrow s$ **46.** $p \leftrightarrow s$

Challenge

47. Writing Use the figures to write a good definition of a *line* in spherical geometry.



Lines Not Lines

48. Multiple Representations You have illustrated true conditional statements with Venn diagrams. You can do the same thing with true biconditionals. Consider the following statement.

An integer is divisible by 10 if and only if its last digit is 0.

a. Write the two conditional statements that make up this biconditional.
b. Illustrate the first conditional from part (a) with a Venn diagram.
c. Illustrate the second conditional from part (a) with a Venn diagram.
d. Combine your two Venn diagrams from parts (b) and (c) to form a Venn diagram representing the biconditional statement.
e. What must be true of the Venn diagram for any true biconditional statement?
f. Reasoning How does your conclusion in part (e) help to explain why you can write a good definition as a biconditional?

Standardized Test Prep

SAT/ACT

49. Which statement is a good definition?

(A) Rectangles are usually longer than they are wide.
(B) Squares are convex.
(C) Circles have no corners.
(D) Triangles are three-sided polygons.

50. What is the exact area of a circle with a diameter of 6 cm?

(F) 28.27 cm (G) 9π m^2 (H) 36π cm^2 (I) 9π cm^2

Extended Response

51. Consider this true conditional statement.

If you want to buy milk, then you go to the store.

a. Write the converse and determine whether it is *true* or *false*.
b. If the converse is false, give a counterexample to show that it is false. If the converse is true, combine the original statement and its converse as a biconditional.

Mixed Review

Write the converse of each statement. See Lesson 2-2.

52. If you do not sleep enough, then your grades suffer.

53. If you are in the school chorus, then you have a good voice.

54. Reasoning What is the truth value of the contrapositive of a true conditional?

Get Ready! To prepare for Lesson 2-4, do Exercises 55–57.

What are the next two terms in each sequence? See Lesson 2-1.

55. 100, 90, 80, 70, . . . **56.** 2500, 500, 100, 20, . . . **57.** 1, 2, 0, 3, −1, . . .